Calculus Warm Up #11-3

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$(-\csc^2 y)\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\frac{dy}{dx}(1 - \csc^2 y) = 1$

$$\boxed{\frac{dy}{dx} = \frac{1}{1 - \csc^2 y}}$$

$\csc^2\left(\frac{\pi}{2}\right) = \frac{1}{(\sin\frac{\pi}{2})^2} = \frac{1}{1}$

$\frac{dy}{dx} @ \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$= \frac{1}{1 - \csc^2\left(\frac{\pi}{2}\right)}$

$= \frac{1}{1-1}$

$= \frac{1}{0}$

undefined.

AP Rev. # 6 (yellow)

4. b) $\longrightarrow$ slope @ $(1,-1) = 2$

$y+1 = 2(x-1)$

$y = 2x - 3$

$f(1.1) \approx 2(1.1) - 3$

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In Exercises 1–44, find the derivative of the given function.

1. $y = x^2 - \frac{1}{2}\cos x$

3. $y = \frac{1}{x} - 3\sin x$

5. $f(x) = 4\sqrt{x} + 3\cos x$

7. $f(t) = t^2 \sin t$

9. $g(t) = \frac{\cos t}{t}$

11. $y = \tan x - x$

13. $y = (5x)(\csc x)$

$y' = 5x(-\csc x \cot x) + 5\csc x$

$5\csc x(-x\cot x + 1)$

15. $f(\theta) = -\csc\theta - \sin\theta$

$f'(\theta) = \csc\theta\cot\theta - \cos\theta$

$\frac{1}{\sin\theta}\frac{\cos\theta}{\sin\theta} - \cos\theta$

$\frac{\cos\theta}{\sin^2\theta} - \cos\theta$

$\cos\theta(\csc^2\theta - 1)$

$\cos\theta\cot\theta$

17. $g(t) = t^2 \sin t + 2t\cos t - 2\sin t$

In Exercises 1–44, find the derivative of the given function.

19. $f(x) = \sin \pi x \cos \pi x$

23. $y = \cos 3x$

25. $y = 3\tan 4x$

27. $y = \sin \pi x$

29. $y = \frac{1}{4}\sin^2 x$

31. $y = \frac{1}{4}\sin^2 2x$

$y = \frac{1}{4}(\sin 2x)^2$

$y' = \frac{1}{2}\underbrace{(\sin 2x)^1(\cos 2x)(2)}_{\sin 2u}$

$= \frac{1}{2}\sin 4x$

33. $y = \sqrt{\sin x}$

35. $y = \sec^3 2x$

$y = (\sec 2x)^3$

$y' = 3(\sec 2x)^2(\sec 2x \tan 2x)(2)$

$y' = 6\sec^3 2x \tan 2x$

33) $y = (\sin x)^{1/2}$

$y' = \frac{1}{2}(\sin x)^{-1/2}(\cos x)$

$\boxed{= \frac{\cos x}{2\sqrt{\sin x}}} \cdot \frac{\sqrt{\sin x}}{\sqrt{\sin x}}$

$= \frac{\cos x\sqrt{\sin x}}{2\sin x}$

$= \frac{1}{2}\cot x\sqrt{\sin x}$

39. $y = e^x(\sin x + \cos x)$

41. $y = e^{\tan x}$

43. $y = \ln|\tan x|$

39) $e^x(\cos x - \sin x) + e^x(\sin x + \cos x)$

$e^x(\cos x - \sin x + \sin x + \cos x)$

$2e^x \cos x$

8.3 continued: Use implicit differentiation to find $\frac{dy}{dx}$, and evaluate at the given point.

$$(2 \sin x)(\cos y) = 1 \qquad \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$$

$$2\sin x(-\sin y)\frac{dy}{dx} + 2\cos x \cos y = 0$$

$$\frac{dy}{dx}(-2\sin x \sin y) = -2\cos x \cos y$$

$$\frac{dy}{dx} = \frac{-2\cos x \cos y}{-2\sin x \sin y}$$

$$= \frac{\cos x \cos y}{\sin x \sin y}$$

$$= \cot x \cot y$$

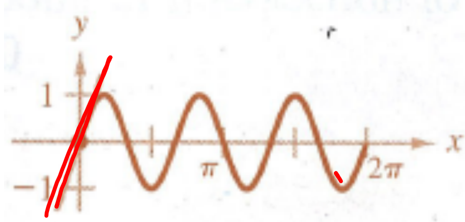
$@\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

$$= \cot\frac{\pi}{4}\cot\frac{\pi}{4}$$

$$= 1$$

Find the slope of the tangent at x = 0, compare this value to the number of complete cycles in the interval [0, 2π].

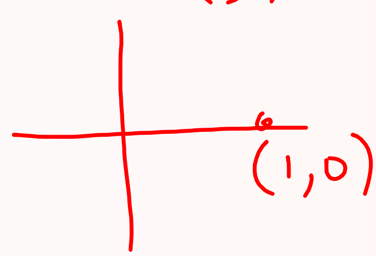
(a) $y = \sin 3x$



$$y' = 3\cos(3x)$$

$$y'(0) = 3\cos 0$$

$$= 1(3)$$



Evaluate the limit. Use L'Hôpital's rule when needed.

$$\lim_{x\to\pi} \frac{\sin x}{x - \pi}$$

$$\lim_{x\to\pi/4} (\tan 2x - \sec 2x)$$

Evaluate the limit. Use L'Hôpital's rule when needed.

$$\lim_{x\to\pi} \frac{\sin x}{x - \pi} = \frac{\sin \pi}{\pi - \pi} = \frac{0}{0} \text{ so L'Hop} \ldots$$

$$= \lim_{x\to\pi} \frac{\cos x}{1} = \cos \pi = \boxed{-1}$$

$$\lim_{x\to\pi/4} (\tan 2x - \sec 2x) = \tan\frac{\pi}{2} - \sec\frac{\pi}{2} = \infty - \infty$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin 2x}{\cos 2x} - \frac{1}{\cos 2x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\sin 2x - 1}{\cos 2x} = \frac{\sin\frac{\pi}{2} - 1}{\cos\frac{\pi}{2}} = \frac{0}{0}$$

Now L'Hop.

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cancel{2}\cos 2x}{-\cancel{2}\sin 2x} = \frac{\cos\frac{\pi}{2}}{-\sin\frac{\pi}{2}} = \frac{0}{-1} = \boxed{0}$$

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45 - 61 odd

Answers to back of slope classwork (tan)

7. D
8. H
9. C
10. F
11. A
12. E
13. B
14. G